Intensity Filter Caused by Cross-Phase Modulation in Dispersion-Managed WDM Systems
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We investigate the influence of cross-phase modulation on a probe channel imposed by a pump channel, in dispersion managed systems. This effect can be described as an intensity filter and the transfer function of the intensity filter for dispersion managed systems is derived, which proves to be a high-pass filter. We investigate some typical dispersion managed systems by using the intensity filter transfer function. We present some useful results that show how to reduce the influence of cross-phase modulation in WDM systems.

1. Introduction
Due to fiber dispersion, phase noise of the optical source can be translated to intensity noise \cite{1}, which directly causes system performance degradation of intensity modulation-direct demodulation (IM-DD) systems. Similarly, phase changes in one channel caused by other channels through cross phase modulation (XPM) introduce intensity noise, which is described by an intensity filter model \cite{2}. Using the intensity filter concept one can expedite the analysis of WDM systems \cite{3}. We extend the intensity filter model into dispersion-managed systems and investigate the intensity filter transfer functions in systems with different dispersion-management schemes. We present some useful results that show how to reduce the influence of XPM by appropriate dispersion management.

2. Theory
We consider a strong pump beam and a relatively weak probe beam in the studied system, which is composed of two segments of fibers with different dispersion coefficients (D1 and D2) and different lengths (L1 and L2, respectively). While the probe beam is a continuous wave the pump beam is a modulated optical signal with the amplitude varying slowly with time. The varying power of pump beam is $P_p(z, t)$, with its Fourier transform $P_p(z, \omega)$. With the loss of pump and the walk-off between pump and probe taken into account, we neglected the SPM of pump and the intensity fluctuation of pump caused by the probe. The pump power can be described in frequency domain as:

$$P_p(z, \omega) = P_p(0, \omega) \exp((-\alpha + j \omega d_{p1})z) \quad 0 < z < L_1 \quad (1)$$

$$P_p(z, \omega) = P_p(0, \omega) \exp(-\alpha z + j \omega d_{p1} L_1 + j \omega d_{p2}(z - L_1)) \quad L_1 < z < L_2 \quad (2)$$

In (1) and (2), \(\alpha\) is the loss coefficient of the optical fiber (which is assumed the same in both segments); the walk-off parameter \(d_{p1} = \frac{1}{v_{gs}} - \frac{1}{v_{gp}} = \int_{\lambda_p}^{\lambda_s} D(\lambda) d\lambda\) can be described here as \(d_{p1} = D_1 \times \Delta \lambda_{sp}, d_{p2} = D_2 \times \Delta \lambda_{sp}\) with \(\Delta \lambda_{sp} = \lambda_s - \lambda_p\). The small phase change of probe beam caused by pump channel during propagation in $dz$ can be described as:

$$P_{p1}(z, \omega) = P_{p1}(0, \omega) \exp((-\alpha + j \omega \Delta \lambda_{sp})z)$$

$$= P_{p1}(0, \omega) \exp((-\alpha z + j \omega \Delta \lambda_{sp} L_1)$$
written as
\[ d\theta_p(z,\omega) = -2\gamma P_p(z,\omega)dz \]  

(3)

In (3), \( r_j = \frac{(2\pi n_j^2)}{\lambda_j A_{\text{eff}}} \) is the nonlinear coefficient with \( n_2 \) the nonlinear refractive index coefficient and \( A_{\text{eff}} \) the effective core area. We assume the same nonlinear coefficient for pump and probe in this study. Due to the fiber dispersion, the phase change is translated into intensity change in the propagation. The intensity fluctuation at the end of the fiber (in frequency domain) caused by \( d\theta_p(z,\omega) \) can be written as [1]:

\[
\frac{dP_p(z,\omega)}{\langle P_s \rangle} = -2\sin \left\{ \frac{\omega^2 \lambda^2}{4\pi c} \left[ D_1(L_1 - z) + D_2 \times L_1 \right] \right\} d\theta_p(z,\omega) \quad 0 < z < L_1
\]

(4)

\[
\frac{dP_p(z,\omega)}{\langle P_s \rangle} = -2\sin \left\{ \frac{\omega^2 \lambda^2}{4\pi c} D_1(L_1 + L_2 - z) \right\} d\theta_p(z,\omega) \quad L_1 < z < L_1 + L_2
\]

(5)

where \( \langle P_s \rangle \) is the average output power of the probe.

The total intensity fluctuation at the end of the fiber can be obtained by integrating \( dP_p(z,\omega) \)

\[
\Delta P_p(\omega) = \int_0^{L_1 + L_2} dP_p(z,\omega)
\]

(6)

After some algebra, we get

\[
\frac{\Delta P_p(\omega)}{\langle P_s \rangle} = P_p(0,\omega) H_p(\omega)
\]

(7)

where

\[
H_p(\omega) = 4\pi \left\{ 1 - \exp(-\alpha L_1 + j\omega d_{sp} L_1 + j\frac{\omega^2 \lambda^2}{4\pi c} D_1 L_1) \right\}
\]

\[
\times \left[ \frac{\omega^2 \lambda^2}{4\pi c} \right]^{-1} \left\{ \alpha - j(\omega d_{sp} L_1 + \frac{\omega^2 \lambda^2}{4\pi c} D_1) \right\} \exp(j\omega^2 \lambda^2 (D_1 L_1 + D_2 L_2)/4\pi c)
\]

(8)

is the transfer function of the intensity filter caused by XPM. \( I_m \{ A \} \) represents the imaginary part of the \( A \). From (7), we can see that the intensity fluctuation of the probe can be understood as the pump output signal from a filter, which is characterized by \( H_p(\omega) \). For WDM systems, we can include the XPM influences of all other channels on one specific channel by adding the XPM influences caused by every channel, which can be easily got using this model.

It is noted that in deriving this filter model, several assumptions are made. The XPM and dispersion are divided in the space, which means we assume linear propagation after XPM happened during \( dz \) in \( z \). The self phase modulation and dispersion of pump are ignored. The assumptions are acceptable where XPM plays a great role for the system performance degradation [4].
3. Numerical results

We calculated the intensity filter transfer function under several circumstances to study the influence of dispersion management on XPM effects. The following parameters values were chosen: probe wavelength 1550 nm, channel spacing 0.4 nm, for all kinds of fibers, loss coefficient 0.2 dB/km, nonlinear coefficient $1.31 \times 10^{-3}$ m$^2$/W. We limit frequency range on study in $0 - 5 \times 10^{10}$ Hz, since up to now the single wavelength bit rate in commercial systems is below 40 Gb/s.

In figure 1 we show the transfer function of 80 km single mode fiber with pre-, post- and without dispersion compensation. The filter is a high-pass filter and we can see dispersion compensation can greatly decrease the influence caused by XPM. We are interested in how the dispersion management affects the intensity filter transfer function. In figure 2 and figure 3 the transfer functions under different dispersion compensation ratio in pre- and post-compensation systems are given. It can be drawn that, to suppress the influence of XPM, the dispersion should be overcompensated in pre-compensation systems while under compensated in post-compensation systems.
In figure 4 we compared two dispersion configurations for fully dispersion compensated systems. 80 km dispersion shifted fiber with dispersion value -2 ps/nm.km is fully compensated by single mode fiber (with dispersion 16 ps/nm.km); 80 km single mode fiber with dispersion value 16 ps/nm.km is fully compensated by dispersion compensation fiber (with dispersion 96 ps/nm.km). We can see that due to the large local dispersion of the single mode fiber, the influence of XPM is suppressed so single mode dispersion is an optimal choice for high-speed WDM systems.

In multi-span WDM systems, the influences of XPM are accumulated in the system end. Figure 6 confirms that. In each span, single mode fiber is compensated by following dispersion compensated fiber completely and the line loss is compensated by the optical amplifier. In figure 7, the under- and over-compensation are studied in a 5-span WDM system. We can see that, on contrast to the single span system, over- and under-compensation both decrease the influence of XPM in multi-span systems.

4. Conclusion

In this letter we take the influence of XPM in WDM systems as an intensity filter and the transfer function of the intensity filter is derived in dispersion managed systems. Its high-pass characteristics can be seen clearly, which means XPM is a dominating nonlinear effect in high-speed WDM systems. Several useful results regarding the optimal dispersion compensation are got, which are useful for the system design.

References