Global Bistability in a Semiconductor Laser with Filtered Optical Feedback

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Through simulations based on the rate equations for a diode laser with filtered external optical feedback, we show that the laser's dynamical system attractors can be controlled through the filter parameters: the filter's spectral width and its central frequency. This is illustrated for a filter-induced global bistability.

Introduction

Introducing small amounts of feedback, say -50 dB, into a solitary diode laser is well known to “destabilize” the output. Underneath these instabilities lies a highly structured complex deterministic dynamics, which may find advantageous applications such as chaotic encryption, pulse generation, optical signal processing or wavelength switching, if sufficient control over the various different types of dynamics can be achieved.

The complexity of diode laser instabilities and dynamics, not just due to feedback, but also due to optical injection or modulation of parameters, e.g. injection current, has led many workers to develop a dynamical description of the laser response (see, e.g. [1-4]). In the context of feedback effects in diode lasers, the dynamical approach has been especially successful in elucidating the mechanisms that are at play in low frequency fluctuation (LFF) phenomena [3], and in understanding the wide range of behaviors that arise in injection lasers [4].

Here we report on our study of filtered optical feedback (FOF) from a dynamical perspective. We look upon the filter as a mechanism for restricting the phase space available to the feedback laser system. During conventional optical feedback (COF), the dynamics of the system occupy a specific region in phase space, which is determined among others by the number of different external cavity modes available (ECM) [5].

Introduction of the filter not only decreases the number of available ECM's, but also moves them around in phase space, which may lead to some unexpected dynamics. Occasionally, for certain parameters, the region of dynamics in phase space is split into two disconnected parts, such that the laser will make a choice about which dynamics it will exhibit (filter-induced global bistability). The filter can also be used to target a desired dynamical behavior (cw, periodic and quasi-periodic oscillations and chaos) in a specific region in phase space.

Model

Our model is for the case of single longitudinal mode operation and consists of three rate equations [6]. The temporal electrical field evolutions in the laser cavity and external system are described by two complex rate equations, the inversion (number of electron-hole pairs) by a real rate equation. We will use the solitary laser frequency $\omega_0$, as the bifurcation parameter since this quantity is directly related to the bias current.
through the phenomenological expression \( \omega_0 = k(J_{\text{thr}} - J) + \omega_{\text{thr}} \), where \( J = I/e \) is the pump rate, \( I \) the bias current, \( e \) the elementary charge, \( J_{\text{thr}} \) the pump rate at threshold operation of the specific longitudinal mode, \( \omega_{\text{thr}} \) the frequency at threshold while \( k > 0 \) is an empirically derived parameter.

As a first step in the analysis the fixed points (CW-solutions to the rate equations, corresponding to monochromatic light emission) are calculated. Figure 1 shows the fixed points in the \((\omega_0, \omega_0(J))\)-plane, where \( \omega \) is the frequency of the compound laser system. For each value of \( \omega_0 \) the intersection of the vertical line through \( \omega_0 \) with the curly curve yields all fixed points at that \( \omega_0 \)-value. In the frequency intervals \((-603, -600)\) and \((-596, -592.5)\) the fixed points are split into two disconnected groups, one corresponding to filter-induced modes (close to the value \( \omega = \omega_f \), indicated by (a) in Fig 1) and the other to a perturbed solitary laser mode (close to the line \( \omega = \omega_0 \), indicated by (b) in Fig 1). It is precisely in these intervals that one should expect filter-induced bistability. This can be studied by investigating the dynamical behavior of the system. Note in this respect, that nothing has been said yet about the stability of the fixed points. In fact, it is well known that feedback has a large destabilizing influence on the system in general, but in the case of filtering one may expect some type of stabilization to occur.

**Figure 1**: The fixed points in the \((\omega_0(J), \omega)\) plane. The line \( \omega_0=\omega_f \) indicates the filter center. The fixed points are created and annihilated in saddle node bifurcations as the solitary laser frequency is changed. The fixed points in the center of the filter are indicated by (a) while the perturbed solitary laser island is indicated by (b). The gray box indicates the region shown in Figure 2. An inter-section of a vertical line with the curve at a specific \( \omega_0 \) gives all the fixed points at that \( \omega_0 \).

**Dynamics**

To investigate the dynamics, numerical integration of the rate equations is performed using a modified Runge-Kutta method of fourth order. Throughout the simulations the internal laser parameters were fixed, while the solitary laser was biased around 40% above threshold. Note that, this high above the threshold the system is in the coherence collapse regime in case of COF [7]. The results are presented in terms of bifurcation maps where the behavior of the instantaneous frequency of the laser system is depicted as a function of the solitary laser frequency. In order to allow quick overview of all the different types of dynamics that may occur during a full scan of the solitary laser frequency, a visualization technique very similar to that of the Poincaré section representation is employed. A plane is defined in a three dimensional phase space, chosen such that all the fixed points lie as close as possible to it, and at each intersection of the phase space trajectory with this plane, the instantaneous frequency is plotted
(black dots) together with the temporal average of the trajectory (thick black lines) and the fixed points (gray line).

The compound cavity modes introduced due to feedback are asymmetric in power, such that modes with frequencies below the solitary laser frequency (red side) have higher power, while modes with higher frequencies (blue side) have lower power than the power emitted at solitary laser operation. This asymmetry is a consequence of the phase-amplitude coupling introduced by the linewidth enhancement factor. Therefore when the filter is tuned to the red side of the solitary laser, the modes inside the filter profile will have higher power than the island of modes around the solitary laser point, and the laser will gain in power from the feedback by locking to these modes (in Fig. 1, for example, fixed points at (a) would correspond to higher power than those at (b)). On the other hand when the filter is tuned to the blue side of the solitary laser frequency, the modes inside the filter profile will have lower power than the solitary laser island, and the laser power will not profit from locking to the center of the filter. This explains why the filter-induced global bistability is found only at the blue side of the filter profile, as was indeed observed in [8].

**Global bistability**

In general the system will show multi-stable behavior between the ECM's which are about ~100 MHz apart. During the global bistability the distance between the attractors is ~4 GHz. The two attractors are centered on the filter center and the solitary laser island of fixed points respectively. In Figure 2 the bifurcation diagram for the filter-induced global bistable interval is shown (as indicated by the gray box in Figure 1). Going from right to left in Fig 2 (increasing pump current) the system is initially in a state of self-oscillations (limit cycle) indicated by (b). When the locking range of the filter is reached, the global bistability appears. In this specific case, the bistability
is between a state of CW-operation at the center of the filter (indicated by (a) in Fig 2) and a state of self-oscillations (b). As one moves closer to the filter center, the solitary laser limit cycle (b) disappears while the CW-state at the center of the filter undergoes a Hopf bifurcation and becomes a limitcycle. Generally the attractor at the center of the filter periodically changes its stability as the solitary laser frequency is scanned. As can be seen in Fig 2, the stable regions decrease in size as we approach the center of the filter.

**Discussions**

We have theoretically analyzed the influence of a relatively narrow frequency filter (FWHM=2 GHz) on the deterministic nonlinear dynamics of a semiconductor laser with filtered external optical feedback. We have focused our analysis on a global bistability induced by the filter and located at the high-frequency flank of the filter profile [8]. This bistability allows switching between two stable dynamical attractors operating on different frequencies (wavelengths), one of which is a self-pulsating mode of operation originating from the solitary laser mode and the other of which is a CW state of operation very close to the filter center frequency. An attractive property of this kind of wavelength bistability is that it can be controlled externally: it is based on the nonlinearity of the external filter, rather than on the intrinsic nonlinearity of the laser.

It should be mentioned that in our numerical experiments we have observed many more interesting dynamical attractors, like limit cycles, quasi-periodicity, and strange attractors, which are not addressed in this paper. An overview of most of the dynamical attractors observed when scanning the solitary laser frequency all the way through the filter profile can be found in [9].

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**References**